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RIGID-PLASTIC MODEL OF CHIPPING DURING METAL CUTTING

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A rigid-plastic model of shearing chip formation is proposed. The model is based on the Mohr–Coulomb yield criterion and allows one to determine the size of chipped elements in orthogonal cutting of plastic and brittle metals.

Key words: *metal cutting, shearing chip, plasticity, Mohr–Coulomb yield criterion.*

An analysis of the most popular theories of metal cutting [1–6] shows that they all provide poor agreement of theoretical calculations with experimental results, except for some particular cases in a narrow range of cutting conditions. At the same time, an advantage of all metal-cutting theories is the improvement of our understanding of the cutting mechanism, thus facilitating the search for effective research methods and new solutions of the problems of metal-cutting mechanics.

A mechanical model based on simple engineering formulas is constructed in the present work for determining the size of the shearing elements and the force of resistance to metal cutting. Shearing of the chip element occurs in the plane that forms an angle Φ (shearing angle) with the direction of cutter motion (Fig. 1). In the present paper, we consider orthogonal cutting with the cutter shaped as a dihedral wedge moving at a right angle to the cutting face. The frontal angle of the cutter formed by the working face of the wedge and the vertical direction (Fig. 1) is denoted by α. The chip OABC enclosed between the frontal face of the wedge and the shearing plane overcomes cohesion forces and moves upward over the shearing plane OA under the action of the resultant force P applied by the wedge. We assume that the friction on the cutter–chip contact area is determined with sufficient accuracy by the mean angle of friction β.

As the resultant of all forces P applied by the working face of the wedge onto the chip forms the angle β with the direction of the normal to this face, the problem is equivalent to the problem where the frontal face of the wedge is ideally smooth and the frontal angle of the cutter is $\gamma = \alpha - \beta$. A similar idea was used in [7] in solving problems of penetration of wedge-shaped stamps with formation of rigid incrustation. Note, the transition to the equivalent problem substantially simplified the search for new rigid-plastic solutions and characteristics of the stress field in the chip on the surface of its contact with the cutter, because the principal stress axes are readily determined in this case. Indeed, as the wedge in the equivalent problem is smooth, the shear stresses in the chip at the line of its contact with the cutter equal zero; hence, the principal stress axes coincide with the directions of the tangent and normal to the frontal face of the wedge.

Thus, the prescribed parameters in the problem considered are the frontal angle of the cutter α, the friction angle β, and the depth a to which metal cutting is performed (see Fig. 1). The cutter width is normally much greater than the depth a; therefore, the deformation can be actually assumed to be two-dimensional.

Depending on physical and mechanical properties of the material processed, cutting mode, cutter geometry, and lubricants and coolants used, the cutting process yields shearing chips, continuous chips, or discontinuous chips [2, 3].

Discontinuous chips are formed in processing brittle and plastic materials with very low cutting velocities and high feeding speeds. In this case, shearing occurs in the strain zone, and the chip is separated into individual elements [2, 3].

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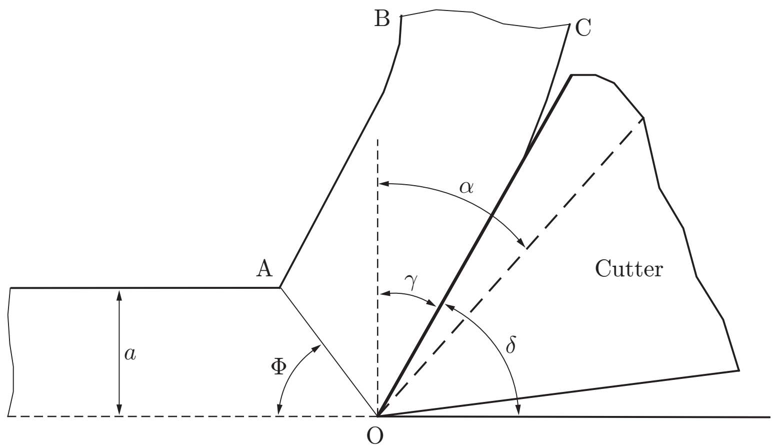

Fig. 1. Model of orthogonal cutting with the shearing plane OA.

Processing of steels and many other viscous materials with moderate cutting velocities, a fairly thick layer being removed by instruments with comparatively small frontal angles, leads to formation of shearing chips consisting of clearly visible separate elements that retain connection with each other. Continuous ships are formed as the cutting thickness decreases, the cutting velocity increases, and the frontal angle of the instrument increases. Vice versa, as the cutting thickness increases, the cutting velocity decreases, and the frontal angle decreases, the chipping process involves separation of chip elements from the billet [3].

In cutting of brittle metals, e.g., cast iron, with certain velocities of cutting by instruments with moderate frontal angles and small cutting depth, the chipping process can proceed in the same manner as that in processing plastic metals.

The above-made analysis of the chipping process offers solid grounds to use plasticity conditions with allowance for metal brittleness in studying the cutting processes [8, 9]. As such a condition, we use the Mohr–Coulomb criterion, where this property is taken into account with the help of the internal friction angle:

$$\max_n |\tau_n| + \sigma_n \tan \varphi = C. \tag{1}$$

Here τ_n and σ_n are the shear and normal stresses on the limiting plane, φ is the angle of internal friction, and C is the cohesion. In Eq. (1), the compressive and tensile stresses are assumed to be negative and positive, respectively.

The rigid-plastic analysis of the cutting process is based on the assumption that there exists a unique shearing plane (isolated slipping line) where the Mohr–Coulomb yield condition is satisfied. Let us analyze forces acting on the chips from the direction of the chip-cutter contact surface and from the shearing line. In what follows, all forces are normalized to the cutting width b. Some elements of this analysis are considered in [1–6]. The resultant of reaction forces R acting on the shearing line OA from the billet on the chip OABC is presented as the sum of the normal R_n and tangent R_t components, which can be determined via the normal and tangent stresses σ_n, τ_n and cohesion C:

$$R_t = \tau_n l, \qquad R_n = \sigma_n l, \qquad R_c = Cl \tag{2}$$

($l = \text{OA}$ is the length of the shearing line and R_c is the cohesion force acting along the shearing line per unit cutting width b). Using Eqs. (1) and (2), we find $R_t = -R_n \tan \varphi + R_c$. Writing the equations of equilibrium in projections onto the n and t axes, we obtain

$$R_t = P \cos(\Phi - \gamma), \qquad R_n = -P \sin(\Phi - \gamma), \qquad P = \frac{Ca \cos\varphi}{\sin\Phi \cos(\Phi - \gamma + \varphi)}. \tag{3}$$

As is seen from Eq. (3), the value of the resultant of pressure forces acting on the chips from the working surface of the cutter depends on the shearing plane orientation characterized by the angle Φ, i.e., $P = P(\Phi)$. Equating the derivative of this function to zero, we find the value of Φ [1]

$$\Phi = \pi/4 + (\alpha - \beta - \varphi)/2 = \pi/2 - (\delta + \varphi)/2,$$

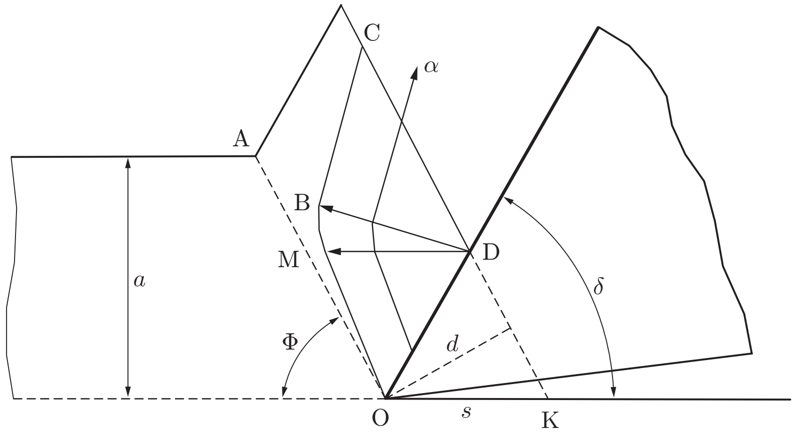

Fig. 2. Rigid-plastic model of chipping during metal cutting.

at which the cutting force reaches a minimum $P = P_\Phi$:

$$P_\Phi = \frac{2Ca\cos\varphi}{1-\sin(\beta-\alpha)} = \frac{Ca\cos\varphi}{\cos^2(\delta/2+\varphi/2)} \tag{4}$$

($\delta = \pi/2 - \gamma = \pi/2 - \alpha + \beta$ is the cutting angle of an equivalent wedge with an ideally smooth frontal face). Then, using Eqs. (2) and (3) and the formula

$$\Phi - \gamma = \pi/4 + (\beta - \alpha - \varphi)/2 = (\delta - \varphi)/2,$$

we determine the shear and normal stresses at the shearing line:

$$\tau_n = C\cos\varphi\,\frac{\cos((\delta-\varphi)/2)}{\cos((\delta+\varphi)/2)}, \qquad \sigma_n = -C\cos\varphi\,\frac{\sin((\delta-\varphi)/2)}{\cos((\delta+\varphi)/2)}.$$

As was noted above, the formation of metallic chips consists of a number of repeated processes with shearing of elements of a certain size. The cutting force is a periodic function. This force increases until the plastic domain expands to the entire thickness of the layer being cut between the frontal face of the wedge and the free surface of the billet. In other words, the cutting force P reaches a maximum value when the shearing plane starts to form and then decreases to a minimum value. Thus, the force of metal resistance to cutting by a flat wedge oscillates with a certain amplitude and frequency. The frequency of oscillations of the resistance force v/s depends on the translational velocity v of wedge motion and on the length s of the shearing element, the latter being the path covered by the wedge during one act of chipping. The distance between the shearing lines d (thickness of the shearing element) is expressed in terms of the length of the shearing element s (Fig. 2) by the formula $d = s\sin\Phi$.

After the shearing plane is formed, the cutting force P decreases to a minimum value. Further motion of the wedge to a distance $s = \mathrm{OK}$ again increases the cutting force to the maximum value (4) and leads to formation of a new shearing plane OA (Fig. 2).

We choose the coordinate system n, t in the following manner: the direction of the t axis coincides with the direction of the frontal face of the wedge, and the n axis is perpendicular to it. Let us formulate the boundary conditions for the problem considered. The OD segment of the metal chip experiences the pressure q from the wedge. As the wedge with the frontal angle $\gamma = \alpha - \beta$ in the equivalent problem is ideally smooth, we obtain $\sigma_n = -q$ and $\tau_{nt} = 0$ on the OD segment.

In formulating the boundary conditions on DC, we choose the coordinate system n, t, such that the t axis is directed along DC, and the n axis is orthogonal to it. If a free surface is formed during shearing-plane formation, i.e., there is no strength relation between the shearing elements, we have $\sigma_n = 0$ and $\tau_{nt} = 0$ on DC. Using these boundary conditions and expressions for the ultimate stresses σ_n and σ_t in the plastic state on DC, we obtain

$$\sigma_n = 0, \qquad \sigma = -C\,\frac{\cos\varphi}{1-\sin\varphi}, \qquad \sigma_t = -2C\,\frac{\cos\varphi}{1-\sin\varphi}.$$

Such a character of chipping, where the material layer being cut is separated by shearing lines into individual elements, is observed in cutting brittle metals. If individual elements of the shearing chip retain the strength relation between them, the ultimate stresses on DC can be represented in the form

$$\sigma_n = -(m-1)C\cos\varphi, \qquad \sigma_t = -\Big(m\,\frac{1+\sin\varphi}{1-\sin\varphi}+1\Big)C\cos\varphi, \qquad \sigma = -mC\,\frac{\cos\varphi}{1-\sin\varphi}, \qquad \tau_{nt}=0. \tag{5}$$

If we use $m = 1$ in these relations, we obtain a free surface on DC. For $m > 1$, all stresses in (5) are compressive, which corresponds, apparently, to formation of continuous chips. We can assume that the case $0 \leqslant m \leqslant 1$ corresponds to formation of shearing chips with retaining connection between the shearing elements. For example, for $m = 0$, we have $\sigma_n = -\sigma_t > 0$ on DC, which corresponds to shearing in the direction of the action of the maximum shear stress.

We solve the problem considered by the method of characteristics, first, for plastic metals with $\varphi = 0$. We assume that the domain of plastic strains OMBCD consists of three subdomains: uniform stress fields OMD and BCD and centered field of slipping lines MBD (Fig. 2). Denoting the mean normal stress by σ and the angle between the slipping line α and the n axis (normal to the frontal face of the wedge; see Fig. 2) by θ, we obtain the following relation along the line α:

$$\sigma - 2C\theta = \text{const}.$$

Using the boundary conditions on OD and DC, we obtain

$$\sigma - 2C\pi/4 = -m\sigma - 2C(\pi/4 + \delta/2).$$

As $\sigma = -q + C$, we have $q = C(m + 1 + \delta)$. The resultant load P_d produced by the frontal face of the wedge is described by the formula

$$P_d = q\,\mathrm{OD} = C(m+1+\delta)d/\cos(\delta/2). \tag{6}$$

Formation of the shearing line OA proceeds as follows. After shearing along the line DC, the cutter starts to cut the next chip under the action of the force P_d, overcoming the resistance of the material processed to plastic strains. As the cutter penetrates inward the billet, the contact area characterized by the segment $l_* = \mathrm{OD}$ increases, which leads to an increase in the force P_d. The contact area is proportional to the length of the segment l_* or to the distance between the shearing lines $d = l_* \cos(\delta/2)$ for a constant cutting angle δ.

During the wedge motion, as is seen from Fig. 2, the distance d between the shearing line CD and the line OA parallel to CD and passing through the apex of the moving wedge permanently increases; in this case, Eq. (6) predicts that the load P_d increases. This process continues until P_d reaches a value equal to the cutting force P_Φ determined by Eq. (4). Thus, the condition of formation of a new shearing line $P_d = P_\Phi$ allows us to determine d:

$$d = \frac{a}{(m+1+\delta)\cos(\delta/2)}. \tag{7}$$

After the ultimate load P_d reaches the value P_Φ, responsible for shearing along the shearing plane OA, further plastic deformation by the scheme shown in Fig. 2 becomes impossible, because it requires an increase in the cutting force. Yet, as this force is sufficient for the shearing plane to form, the chip element formed will be shifted along the plane composing the angle $\Phi = \pi/2 - (\delta + \varphi)/2$ to the cutting direction.

From Eq. (7), we find the length of the contact line l_*:

$$l_* = \frac{d}{\cos(\delta/2)} = \frac{a}{(m+1+\delta)\cos^2(\delta/2)}.$$

The length of the shearing line l and the ratio l/l_* for plastic metals are determined by the formulas

$$l = a/\cos(\delta/2), \qquad l/l_* = (m+1+\delta)\cos(\delta/2). \tag{8}$$

Let us now consider cutting of brittle metals with $\varphi > 0$. We come back to the results of [9], where the problem with plastic domains shown in Fig. 2 was solved. We write the relation valid along the slipping line α:

$$\cot\varphi \ln(1 - (\sigma/C)\tan\varphi) + 2\theta = \text{const}.$$

Using the boundary conditions on OD and DC in the form (5), we obtain

$$\cot\varphi \ln\Big(1 - \frac{\sigma}{C}\tan\varphi\Big) + 2\Big(\frac{\pi}{4} - \frac{\varphi}{2}\Big) = \cot\varphi \ln\Big(\frac{1+(m-1)\sin\varphi}{1-\sin\varphi}\Big) + 2\Big(\frac{\pi}{4} - \frac{\varphi}{2} + \frac{\delta-\varphi}{2}\Big).$$

Solving this equation with respect to σ, we find

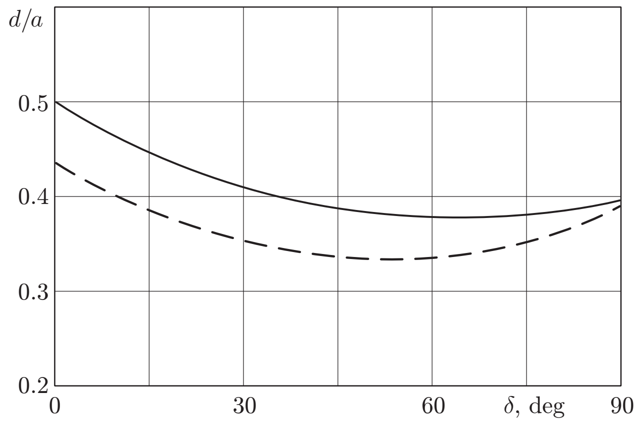

Fig. 3. Size of the shearing element versus the cutting angle for two different angles of internal friction φ: the solid and dashed curves refer to $\varphi = 0$ and $20°$, respectively.

$$\sigma = C \cot \varphi \Big(1 - \frac{1 + (m-1)\sin\varphi}{1 - \sin\varphi}\, e^{(\delta - \varphi)\tan\varphi}\Big). \tag{9}$$

Then, using the boundary condition on OD, we determine

$$\sigma = -\frac{q}{1 + \sin\varphi} + \frac{C\cos\varphi}{1 + \sin\varphi}. \tag{10}$$

Substituting Eq. (10) into (9), we find q and the ultimate load $P_d = q\,\mathrm{OD} = qd/\cos(\delta/2 - \varphi/2)$. As a result, we obtain

$$P_d = \frac{Cd\cot\varphi}{\cos(\delta/2 - \varphi/2)} \Big[(1 + \sin\varphi)\,\frac{1 + (m-1)\sin\varphi}{1 - \sin\varphi}\, e^{(\delta-\varphi)\tan\varphi} - 1\Big]. \tag{11}$$

Equating this value of the force P_d to the cutting force P_Φ found from Eq. (4), we find the size of the shearing element, i.e., its thickness d:

$$d = \frac{a\sin\varphi\cos(\delta/2 - \varphi/2)}{\cos^2(\delta/2 + \varphi/2)\Big[(1 + \sin\varphi)\,\dfrac{1 + (m-1)\sin\varphi}{1 - \sin\varphi}\, e^{(\delta-\varphi)\tan\varphi} - 1\Big]}. \tag{12}$$

The formula obtained allows us to determine the size of the shearing element with known physical and mechanical properties of the metal, thickness of the removed layer a, and cutting angle δ. Figure 3 shows the normalized size of the shearing element d/a as a function of the cutting angle δ for two different angles of internal friction φ. Knowing the size of the shearing element d, we determine the length of the contact line l_* and the length of the shearing line l by the formulas

$$l_* = d/\cos(\delta/2 - \varphi/2), \qquad l = a/\cos(\delta/2 + \varphi/2).$$

Then, we find the ratio of the shearing area lb to the contact area $l_* b$:

$$\frac{l}{l_*} = \frac{\cos(\delta/2 + \varphi/2)}{\sin\varphi}\Big[(1 + \sin\varphi)\,\frac{1 + (m-1)\sin\varphi}{1 - \sin\varphi}\, e^{(\delta-\varphi)\tan\varphi} - 1\Big]. \tag{13}$$

The length of the contact line of the frontal face of the cutter with the chips l_*, which has been identified with the crushing line $l_0 = \mathrm{OD}$ up to now, satisfies the inequality $l_* < l_0$ in the case of formation of shearing chips both for plastic [2] and brittle metals [3]. The measurement of the size l_0 of chipping elements [3] obtained by low-velocity cutting shows that we have

$$l/l_0 = 1.5\text{–}2.0 \tag{14}$$

for steel, iron, and viscous bronze with cutting angles of 45–75° and

$$l/l_0 = 2\text{–}3 \tag{15}$$

for hard bronze and cast iron.

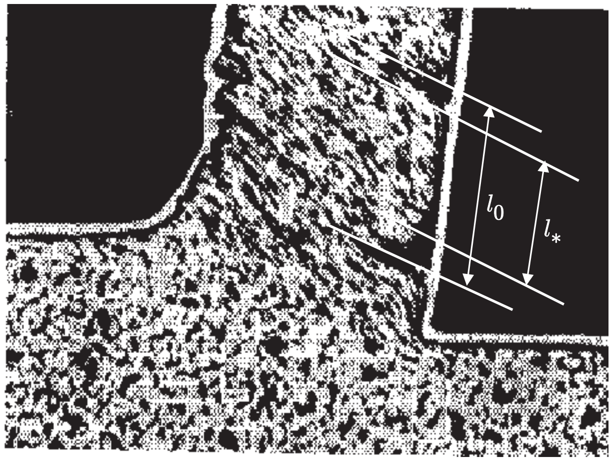

Fig. 4. Microphotograph of the cross section of the chip foot registered in cutting of 20Kh steel [2].

We determine the size of the shearing elements by formulas (8) and (13) for plastic and brittle metals. As was noted above, the real area of the element–cutter contact l_*b is slightly smaller than the crushing area l_0b; hence, we can write $l_* = \lambda l_0$, where $\lambda \leqslant 1$, which is the case observed on the photographs [2, 3] for brittle and plastic metals with formation of shearing chips. Figure 4 shows the photograph of the cross section of the chip foot registered in free cutting of 20Kh steel ($\alpha = 10°$, $b = 10$ mm, and $v = 0.7$ m/min) [2].

In further calculations, we assume that the cutting angle is $\delta = 75°$, which corresponds to the mean values of this angle in experiments with allowance for chip friction on the cutter [3]. We have $\varphi = 0$ for steel, iron, and viscous bronze and $\varphi = 20°$ for hard bronze and cast iron. If we use $m = 1$ and $\lambda = 0.75$ in Eq. (8), we obtain $l/l_0 = 1.98$. If we use $m = 1$ and $\lambda = 0.75$ in Eq. (13), we find $l/l_0 = 2.8$. In each case, the shearing element was assumed to become completely separated from the chip, i.e., $m = 1$. Similar values of l/l_0 can be calculated for shearing elements retaining their connection with the chip ($m \neq 1$) and for those whose contact area coincides with the crushing area ($\lambda = 1$). As both facts indicated above are observed in reality, we can argue that the calculations by formulas (8) and (13) agree with experimental results both for plastic (14) and for brittle (15) metals.

Thus, the rigid-plastic model of shearing chip formation, suggested in the present paper, allows one to use Eq. (12) to determine the size of the shearing element, which depends on the cutting depth, angle of internal friction of the metal, frontal angle of the cutter, and mean angle of friction of the chips on the cutter. The calculated results are in reasonable agreement with experimental data.

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